

Quantum Interference of Particles and Resonances

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When looking at the Maxwell equations,
it is hard to imagine how beautiful the rainbow is.

R. P. Feynman (*cited from memory*)

Similar may be said about **Quantum Interference**

Everybody knows
that the interference does exist.
But it is not always easy to imagine
how it will work in a particular case.

Let us begin from the beginning

- **Quantum physics** is probabilistic.
- **Classical physics** can be probabilistic as well (**Statistical physics**).
- **Essential difference:**
 - Classical Phys. adds probabilities;
 - Quantum Phys. adds amplitudes.
- **Important consequence:**
 - possibility of **interference** effects;
 - various wave functions may **mix**.
- **Impressive result:**
 - particles may **oscillate** in time,
 - transforming to each other.

The most famous examples:

- strangeness oscillations –
in decays of neutral K -mesons (at ~ 10 cm),
- beauty oscillations –
in decays of neutral B -mesons (at some μm)
[coherent oscillations of several flavors are also possible
Ya.A., PR D42 (1990); EPJ A4 (1999)],
- neutrino oscillations (up to hundreds km , MINOS
or even astronomical distances ! solar ν)
A wide variety of macroscopic distances !

Quite **macroscopic** manifestations of quantum **microscopic** effects !

Let us consider in more detail the time dependence in decays of neutral kaons

Neutral kaons decay as a coherent mixture of the **short**-lived K_S and the **long**-lived K_L .

$$\tau_S = 0.9 \cdot 10^{-10} \text{ s} ;$$
$$\tau_L = 5.1 \cdot 10^{-8} \text{ s}$$

At **small** times, the final state 2π comes mainly from K_S (K_L/K_S amplitude ratio is $2 \cdot 10^{-3}$).

At very **high** times, when K_S died, only K_L provides 2π .

no
inter-
ference

Interference for $K_S, K_L \rightarrow 2\pi$ is well seen only in the transition region, after several τ_S .

Non-universality !

Interference
may depend
on the decay
mode

For **semileptonic decays** the **interference** seen from the initial moment till K_S dies.

$\left\{ \begin{array}{l} K_S, K_L \text{ amplitudes are} \\ \text{nearly the same, up to sign.} \end{array} \right.$

Hadronic resonances may also mix
and oscillate in time -

evolution of $\rho^0, \omega \rightarrow \pi^+ \pi^-$ is similar to $K_S, K_L \rightarrow \pi^+ \pi^-$:

ρ^0 is shorter-lived, decay $\omega \rightarrow \pi^+ \pi^-$ is weaker
(isospin suppression for ω , **CP**-suppression for K_L).

But it is unobservable,

because of too short lifetimes:

$$\tau < 10^{-20} \text{ s}, \quad c\tau < 3 \cdot 10^{-10} \text{ cm}$$

$$[\text{compare } \tau_S = 0.9 \cdot 10^{-10} \text{ s}, \quad c\tau_S = 2.7 \text{ cm}]$$

However, the same phenomenon may be seen in the complementary variable - **energy** (**mass** in the rest frame):

it is seen here as deformation
of Breit-Wigner peaks.

- The pure BW term $|a \cdot (E - E_0 + i \Gamma/2)^{-1}|^2$
 $= |a|^2 \cdot [(E - E_0)^2 + \Gamma^2 / 4]^{-1} ;$

may depend on E

- BW with background $|B + a \cdot (E - E_0 + i \Gamma/2)^{-1}|^2$
 $= |B|^2 + |a|^2 \cdot [(E - E_0)^2 + \Gamma^2 / 4]^{-1}$
 $+ [2|B a| \cdot \cos \varphi \cdot (E - E_0) + |B a| \cdot \sin \varphi \cdot \Gamma]$
 $\times [(E - E_0)^2 + \Gamma^2 / 4]^{-1}$

interference
term

role of the interference depends on the relative value and on the relative phase φ of B and a ; it is **linear** in a , may change **sign** and be either **positive** or **negative**

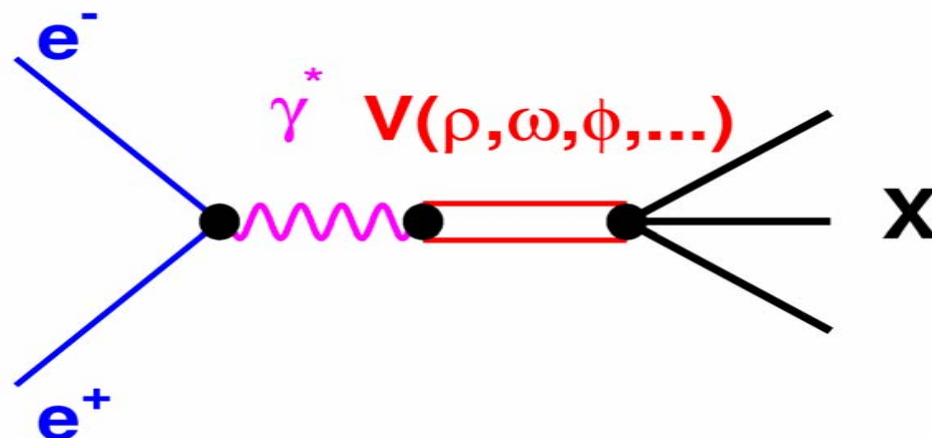
Intermediate conclusions (1)

- At a **small value** of $|a/B|$ the **interference** term may be more essential than the **proper BW** contribution.
- Due to additional **E** -dependence, the **interference** may change sign, provide either **bump**, or **dip**, or both.
- The **bump** and/or **dip** **positions** are, in general, **shifted** from the true position of the resonance.
- The same resonance may interfere differently in different decay modes.

A rich source of examples,
how the **interference** works,
is provided by the reaction

$$e^+ e^- \rightarrow \text{hadrons}$$

Contributions
with the same
final state are
coherent;
they all are
produced
through γ / Z
and may
directly interfere,
if have the same
decay mode



Independent contribution of a resonance
is a BW-peak, proportional to $\Gamma_{ee} \cdot \Gamma_X / \Gamma_{\text{tot}}$;
interference may **change** its form and intensity

$$\Gamma(\rho) = 149.4 \text{ MeV}; \quad \Gamma(\omega) = 8.5 \text{ MeV}; \quad \Gamma(\phi) = 4.3 \text{ MeV}$$

$$\Gamma(\omega \rightarrow 3\pi) = 7.58 \text{ MeV}$$

$$\Gamma(\rho^0 \rightarrow 3\pi) = 0.015 \text{ MeV}$$

$$\Gamma(\phi \rightarrow 3\pi) = 0.65 \text{ MeV}$$

isospin violated
Zweig rule violated

hep-ex/0604051

Bkg near ϕ
changes slowly



nearly standard
interference curve,
instead of ϕ -peak:
both **bump** and **dip**,
each has the **form**
different from BW;
max/min different
from the ϕ -mass

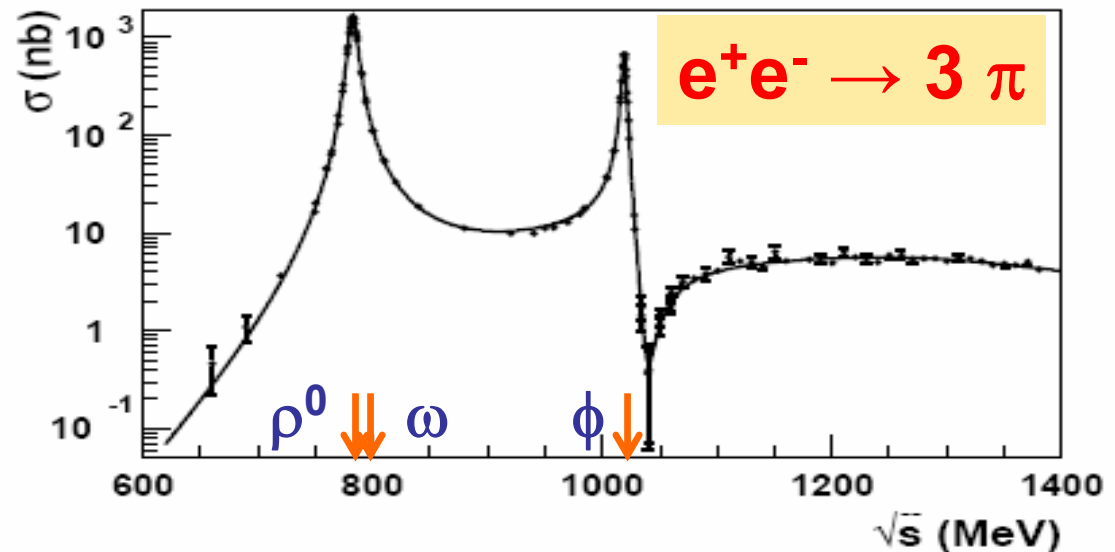


Figure 4. The $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ cross section measured by SND [44,46,34]. The curve is the fit with the $\omega, \phi, \rho, \omega', \omega''$ resonances.

ρ -contribution here deforms ω -tails

(see later)

$$\Gamma(\rho \rightarrow ee) = 7.04 \text{ keV}; \quad \Gamma(\omega \rightarrow ee) = 0.60 \text{ keV}$$

$$\begin{aligned}\Gamma(\rho \rightarrow \eta\gamma) &= 44.9 \text{ keV} \\ \Gamma(\omega \rightarrow \eta\gamma) &= 4.1 \text{ keV} \\ \Gamma(\phi \rightarrow \eta\gamma) &= 55.6 \text{ keV}\end{aligned}$$

← u, d subtraction

$$e^+e^- \rightarrow \eta\gamma$$

hep-ex/0512027

SND Collab.

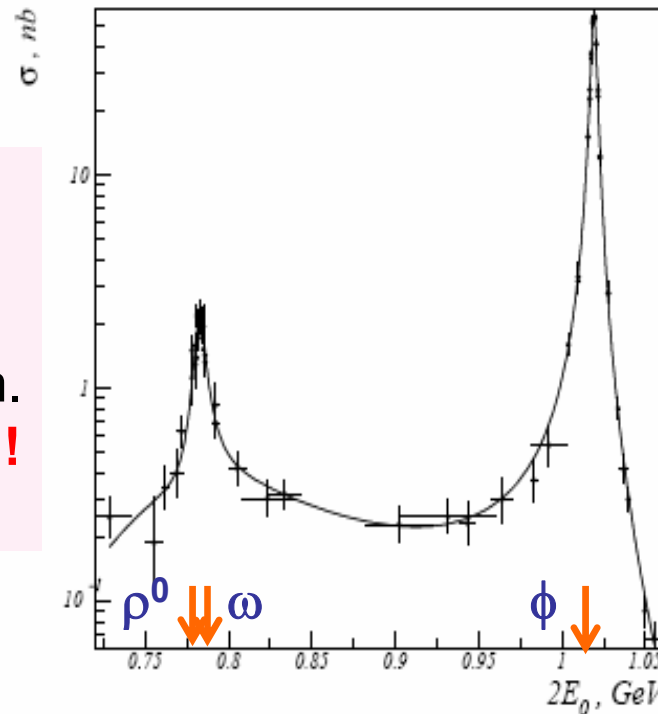


Figure 5: The measured cross section of the $e^+e^- \rightarrow \eta^0\gamma$ process, the curve is the best fit.

The ρ, ω peak has
break near m_ρ ,
while **max** near m_ω ,
despite the smaller ω -term.
Constructive **interference** !
(details see later)

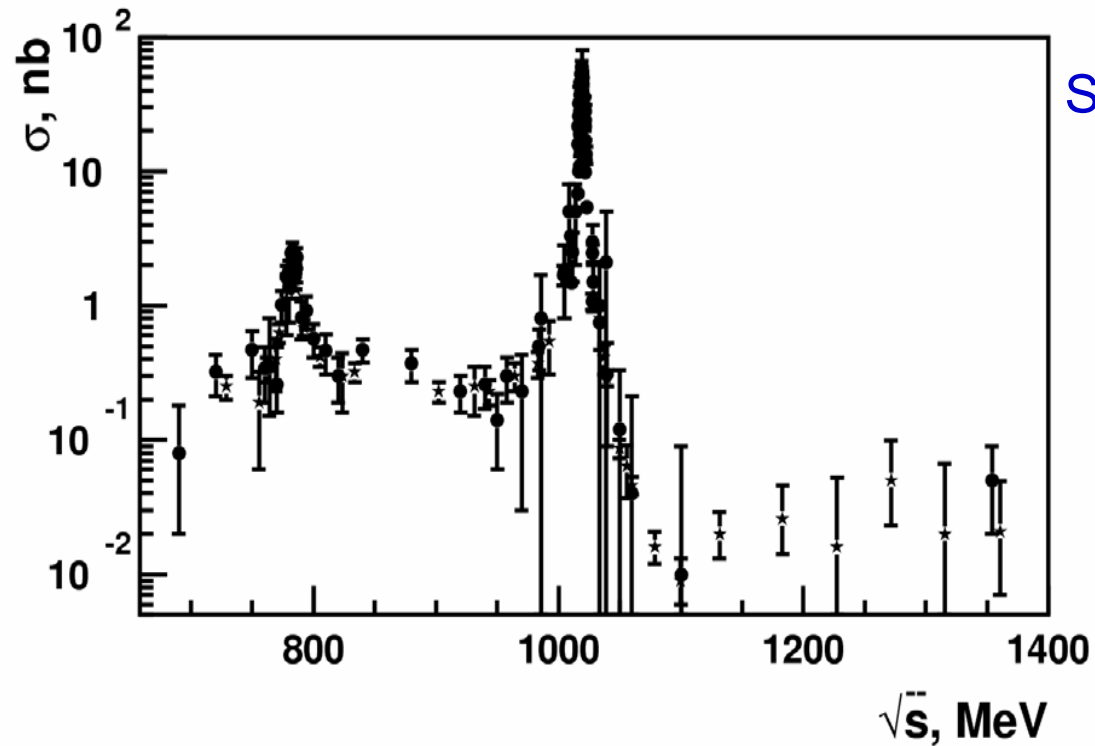
The *right* side
of the ϕ -peak
is **sharper** than
the *left* one.

Interference ?

$$e^+ e^- \rightarrow \eta \gamma$$

hep-ex/0604051

SND, CMD2
Collabs.



Cross sections
at the left and
to the right of the
narrow ϕ -peak
are different.

Interference !

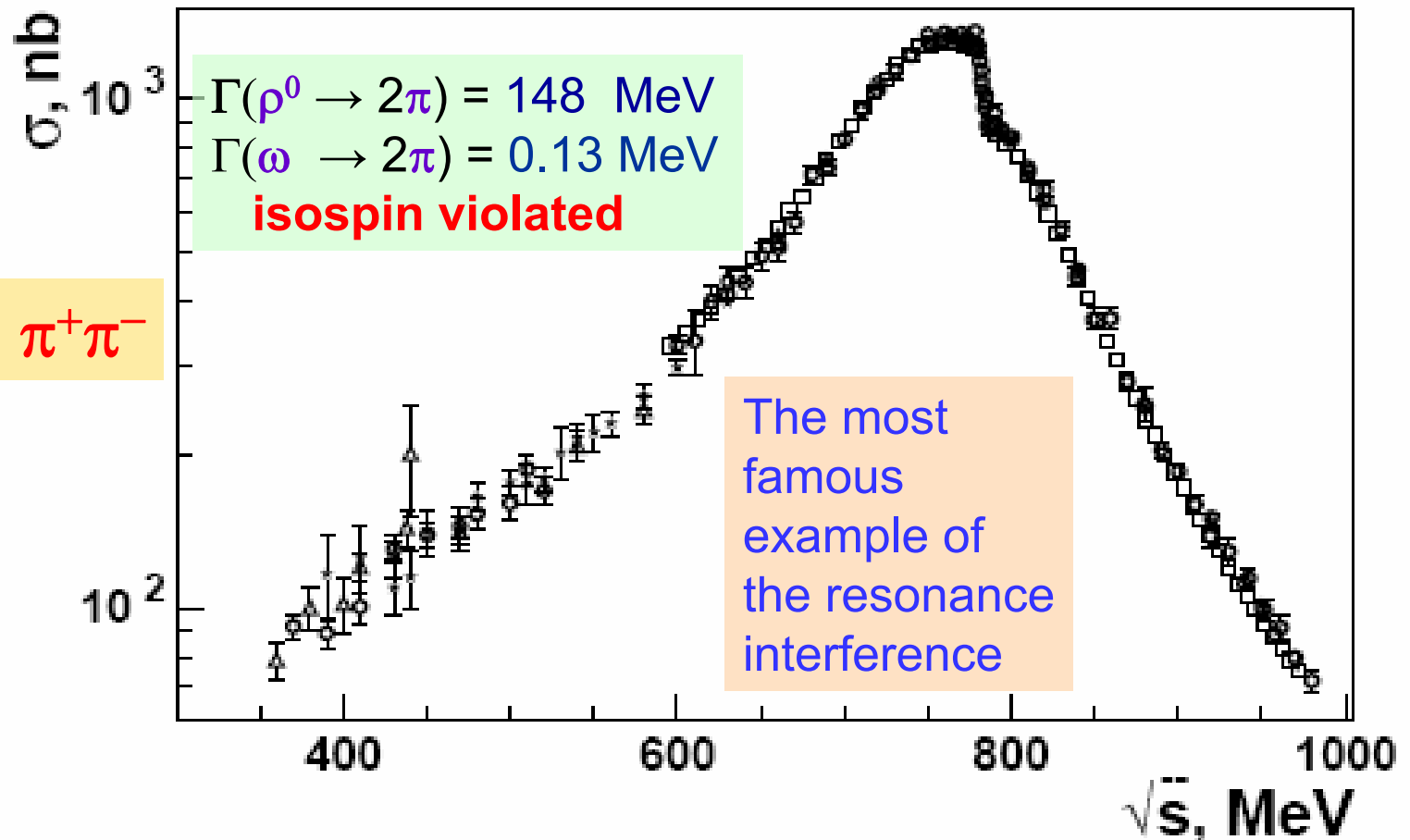
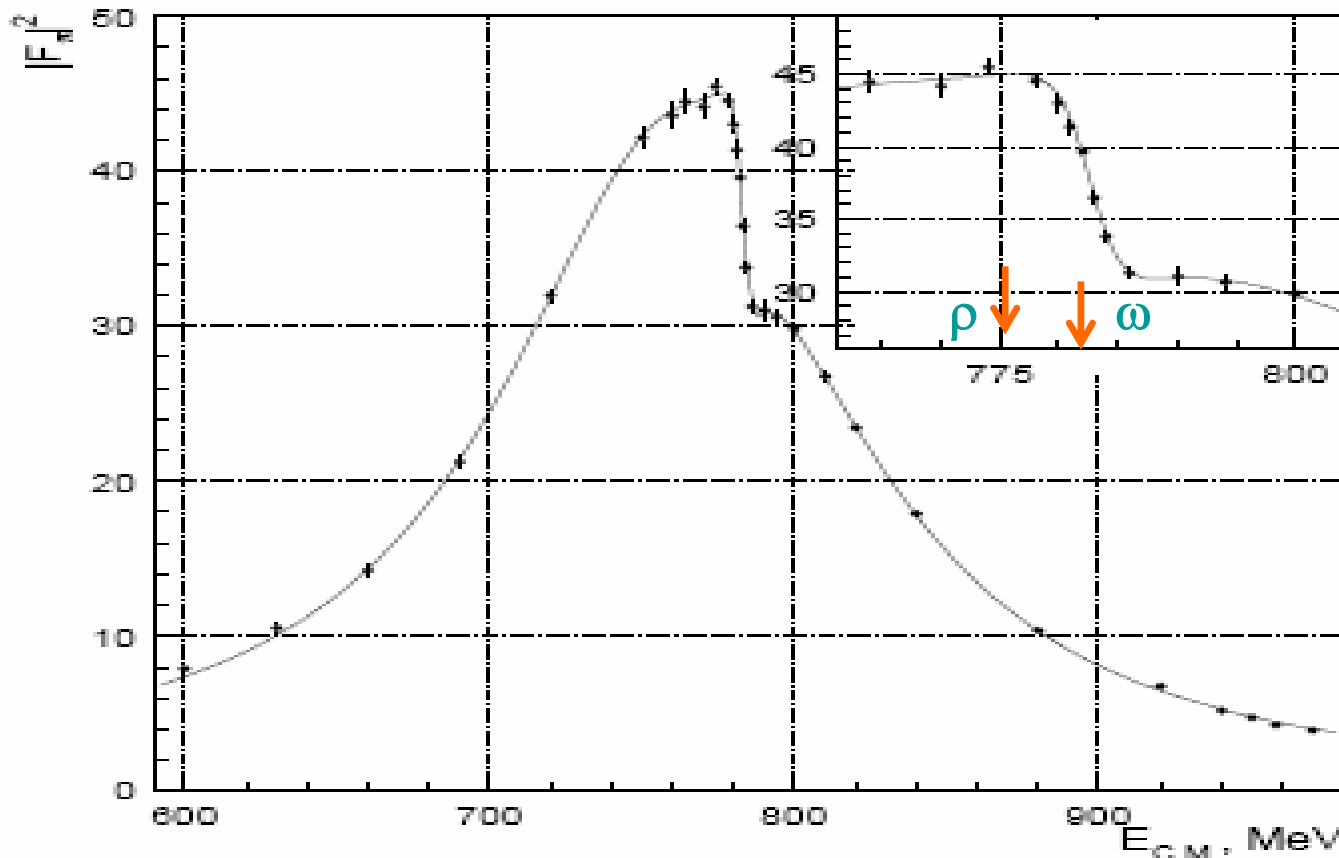


Figure 2. The $e^+e^- \rightarrow \pi^+\pi^-$ cross section. OLYA and CMD (Δ) [36], SND (\star) [37], CMD-2 (\circ) [38,39,40] and KLOE (\square) [5] data are shown.



$$e^+e^- \rightarrow \pi^+\pi^-$$

hep-ex/0610021
CMD-2

interference
is the only
source
of information
on the decay
 $\omega \rightarrow 2\pi$

Figure 9: Fit of the pion form factor measured in this work

Specifics of this case: rapidly **decreasing** “bkg” (ρ –peak) ;
 ρ – ω mixing may (and does) have **complexity** [Ya.A., EPJ A16 (2003)]
 \Rightarrow interference curve is strongly **asymmetric**: decrease, no increase.

The opposite relative sign would reveal additional peak (the case of $\eta\gamma$).

$$\Gamma(\omega \rightarrow \pi^0 \gamma) = 0.76 \text{ MeV}$$

$$\Gamma(\rho^0 \rightarrow \pi^0 \gamma) = 0.09 \text{ MeV} \quad u, d \text{ subtraction}$$

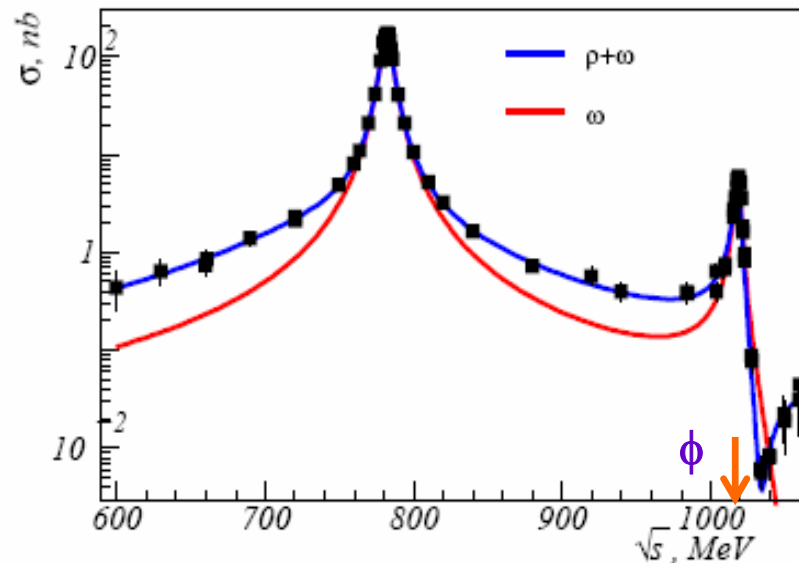
$$\Gamma(\phi \rightarrow \pi^0 \gamma) = 0.005 \text{ MeV} \quad \text{Zweig rule violated}$$

$$e^+e^- \rightarrow \pi^0 \gamma$$

hep-ex/0512027

SND Collab.

Interference
of 3 vector mesons:
 ρ^0, ω, ϕ



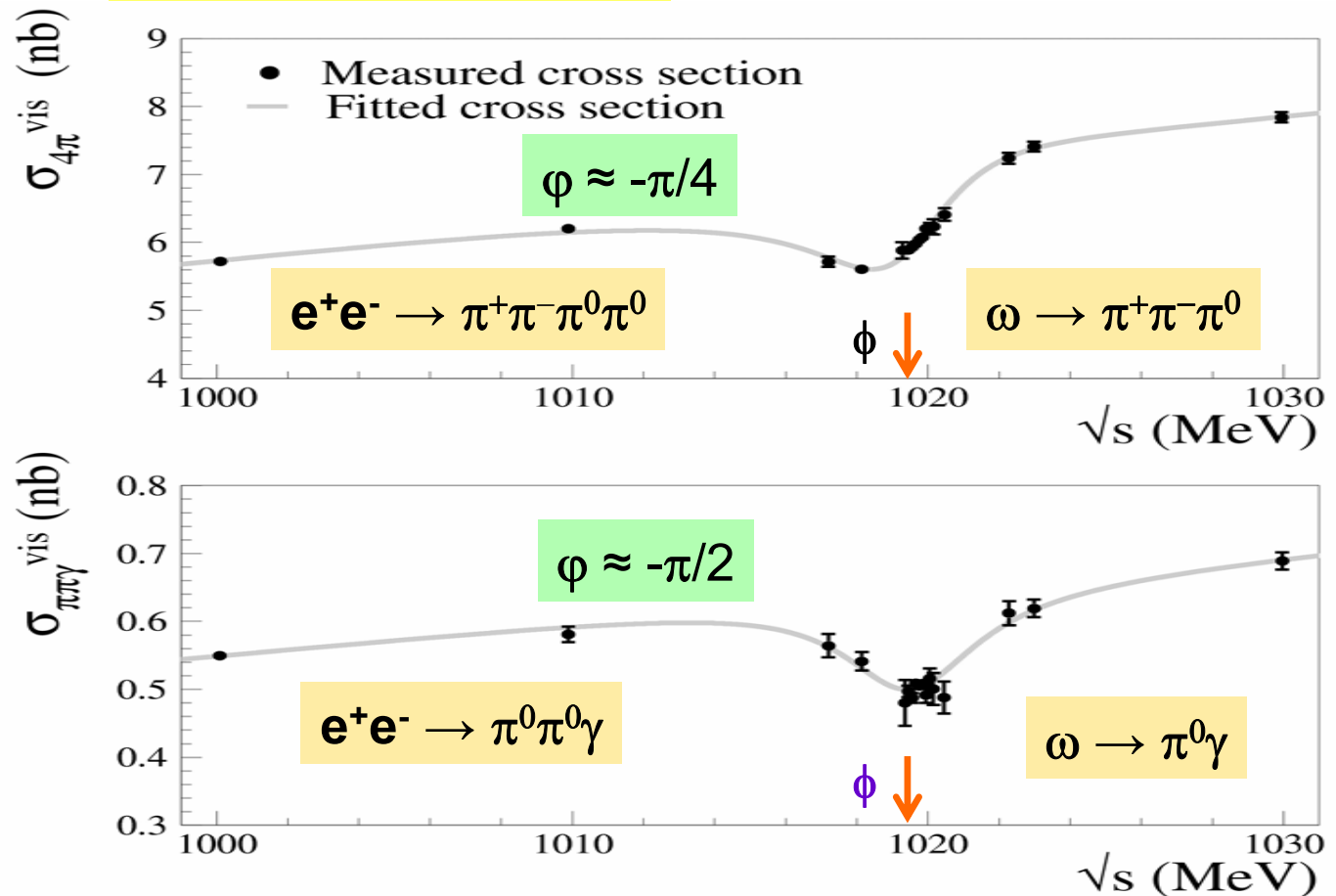
Structure of the
interference curve
is qualitatively
similar here
to the case of 3π

Figure 4: The measured cross section of the $e^+e^- \rightarrow \pi^0 \gamma$ process, two fitting curves correspond to models with $\rho + \omega$ and ω intermediate states respectively.

$\phi \rightarrow \omega \pi^0$
doubly suppressed:
Zweig rule, isospin

arXiv: 0807.4909 [hep-ex]
 KLOE Collab.

The two different curves have different dip positions, because of different relative complexity



These data result in

$$\text{Br}(\phi \rightarrow \omega \pi^0) = (4.4 \pm 0.6) \times 10^{-5} ;$$

$$\Gamma(\phi \rightarrow \omega \pi^0) = 0.19 \text{ keV}$$

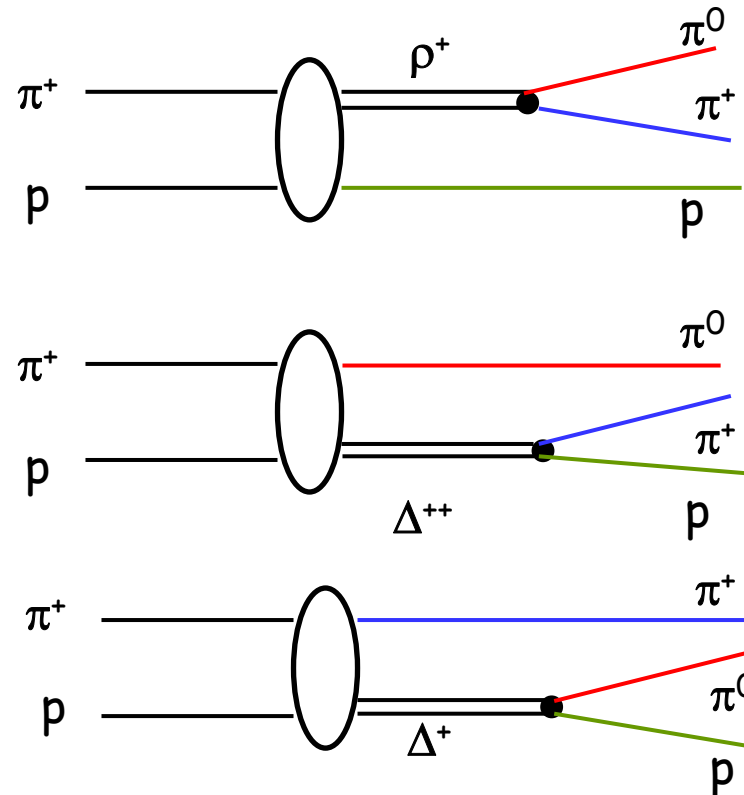
Intermediate conclusions (2)

- All the above examples demonstrate the **direct interference** of 2 resonances:
all final particles can be decay products of any of the interfering resonances.
- Such a kind of **interference** appears very efficient to search for rare decays of known resonances.
May strongly **deflect** a **resonance** manifestation from **the familiar BW peak**.
- There can be **other kinds** of **interference**, where only **some of final particles** may come from **any of the 2 interfering resonances**.

Rescattering interference

Different resonance configurations may produce the **same** state of 3 or more particles. Such contributions are coherent and may **interfere**

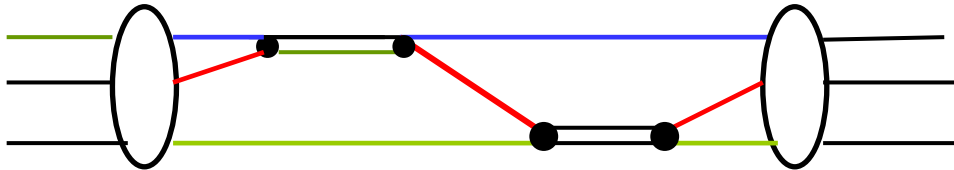
The contributions depend on energies and momentum transfers; may **shift and move** positions of bumps/dips



The phenomenon is known since 60s. It was considered as **hindrance** to resonance studies.

This interference was usually (and is **still now**) cut away.

Rescattering interference



The name **rescattering** reflects similarity with the rescattering in 3-particle interactions: **one particle** changes its interaction partner.

On the other side, this kind of **interference** is similar to the famous case of 2 quantum slits, since **one particle** refers **simultaneously** to **2 resonances**.

The resonances are in different **systems** and may have different **quantum numbers**, but final states, after resonance decays, should contain the same particles.

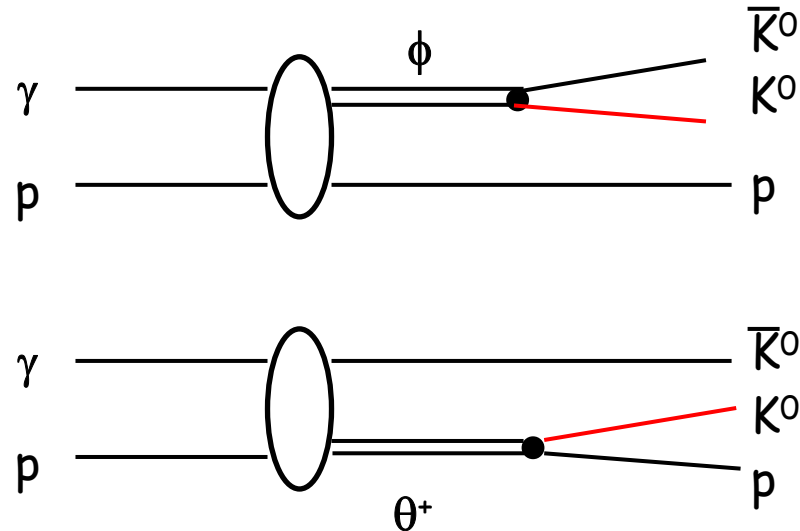
The 2 resonances **can interfere**, **only if** the final configurations are kinematically consistent.



This requires limited intervals of the **total energy** and **momentum transfers**

Positions of the **interference bumps/dips**, generally, depend on kinematic parameters and **move** with their changes (in difference with **true** resonance positions)

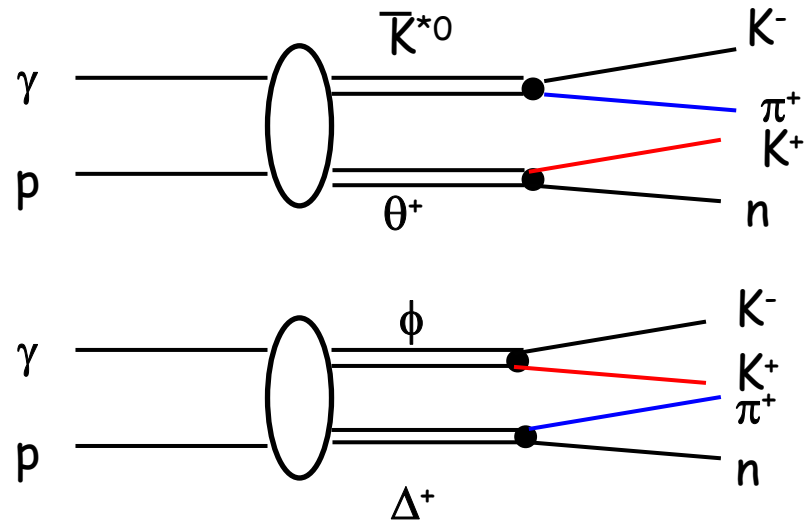
Direct interference of resonances has become an efficient **instrument** actively used to study rare decays of **known** resonances.



Amarian, Diakonov, Polyakov (hep-ph/0612150) have suggested to apply the **rescattering interference** for revealing small **new** resonance signals (**amplification by interference with strong signal**).

The ϕ -signal may indeed be small, if its production is a new kind of hard processes [Ya.A., Goeke, Strakovsky, PR D76 (2007)].

Final states with >3 particles
admit more complicated cases
of the rescattering interference



An example of the 4-particle rescattering interference
that may enhance the small θ^+ -contribution
(suggested by M.Amarian)

Summary

- Interference of resonances (in the energy representation) has the same origin as the known particle oscillations (in the space-time representation).
- A small resonance contribution may be amplified and revealed due to its interference with high background (e.g., another resonance).
- Manifestation of an interfering resonance may be very different: bump, or dip, or both; may depend on the decay mode. Positions of the bump/dip are, in general, shifted from the true position of the resonance.
- The form of the “resonance” curve essentially depends on properties of the background and on the Res-Bkg relative phase: it may be symmetric, or antisymmetric, or strongly asymmetric.

Summary (cont.)

- **Direct** interference is actively used now as an important instrument for resonance studies. Some rare decays of well - established resonances are known **only** due to interference manifestations.
- **Rescattering** interference of resonances may be very useful as well: to amplify small resonance signals, especially with **new quantum numbers**; to study **production mechanisms** of the known resonances.
- **Interference of resonances** looks to be worth of more detailed studies, both **experimental** and **theoretical**.